Fermi-Bose Transmutation for Stringlike Excitations of Maxwell-Higgs Systems

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Abstract

We show that a closed Nielsen-Olesen string in presence of a point scalar source exhibits the phenomenon of Fermi-Bose transmutation. This provides physical support to previous claims about transmutation between bosonic and fermionic one-dimensional structures in (3+1) dimensions. In order to render the computations mathematically rigorous we have resorted to an Euclidean lattice regularization.

Two different approaches have been followed in order to prove the (2+1) Fermi-Bose transmutation. First, Wilczek [1] showed that the bound state of a quantum point charge and a classical vortex can have any (fractional) angular momentum. Secondly, Polyakov [2] proved the fermion-boson transmutation by considering properties of an scalar field interacting with a Chern-Simons term. When the partition function of the scalar particle represented by a Wilson loop is computed, the Chern-Simons action leads to a Gauss linking number. It turns out that the effect of this linking number contribution is to turn the bosonic propagator into a spinorial propagator. In both cases the underlying physical phenomenon which produces the transmutation of bosons into fermions is the Ahronov-Bhom [3] effect.In particular, the statistics of the composite system depends on the value of the product: charge $\times$ magnetic flux.

The two mentioned approaches have been extended to string-like objects in (3+1) dimensions. It was shown [4] that a composite formed from a Nambu charged closed string interacting with a Kalb-Ramond point vortex in a multiply connected three dimensional space [1] can have fractional angular momentum. On the other hand, it was also shown that a naive extension of Polyakov's construction to the (3+1)-dimensional case leads to transmutation between bosonic and fermionic one-dimensional structures [6].

However, the considered systems in (3+1)-dimensions where based on rather *ad hoc* models.In fact, Nambu strings in (3+1)-dimensions are not expected to be fundamental objects but they should rather be considered as effective excitations of some underlying field theory. In this letter we address the issue of supplying the physical realization that was lacking.To do that we shall start by considering a field theory (the Maxwell-Higgs system) whose classical solitonic excitations behaves as a Nambu string interacting with a Kalb Ramond vortex. Then we shall quantize the theory by computing the transition amplitude in terms of the Feynman Path integral, and by making contact with the Polyakov construction we shall prove the Fermi-Bose transmutation for the stringlike excitations of the Maxwell-Higgs system.

[1] A torus was excised from the space manifold in order to provide stability for the closed strings and the point source was located in the forbidden region. A similar construction was used by Bowick et al [5].

The action of the system of ref. [4] was

$$S = S_{NG} + \frac{1}{2}e\int_{\tau_1}^{\tau_2}\int_0^{2\pi} d\sigma B_{\mu\nu}(x)[\dot{x}^\mu x'^\nu - x'^\mu \dot{x}^\nu], \tag{1}$$

$$S_{NG} \propto \int_{\tau_1}^{\tau_2} d\tau \int_0^{2\pi} d\sigma[(x'\dot{x})^2 - x'^2\dot{x}^2]^{1/2}, \tag{2}$$

where S_{NG} is the Nambu-Goto bosonic string free action, $\dot{x}^\mu = \partial^\mu x/\partial\tau$, $x'^\mu = \partial^\mu x/\partial\sigma$ and $B_{\mu\nu}$ is a rank-2 potential given by

$$B_{ij} = Q\epsilon_{ijk}x_k/r^3, \qquad i = 1,2,3, \;\; j = 1,2,3, \tag{3}$$

such that it generates the point vortex of Kalb-Ramond field at the origin:

$$\begin{aligned} H_{ijk} &= Q\epsilon_{ijk}\delta^3(x), \\ H_{0ij} &= 0. \end{aligned} \tag{4}$$

The bosonic action (2) was regarded as an effective action for solitonic solutions of some underlying theory. In order to provide such a field theory our starting point is the well known fact that the $D = 3+1$ Maxwell-Higgs theory, the relativistic generalization of Ginzburg-Landau theory of superconductivity, allows for vortex-line solutions [7], namely, the Nielsen-Olesen strings. Those strings, which are the analogous to the Abrikosov vortex lines in a type II superconductor, can be identified with the Nambu string in the strong coupling limit. Furthermore, the Maxwell-Higgs classical action:

$$\begin{aligned} S = \int \frac{1}{2}F\wedge {}^*F \;\; &+ \;\; \frac{1}{2}(d+ieA)\phi \wedge {}^*(d-ieA)\bar{\phi} - V(\phi) \\ V(\phi) \;\; &= \;\; \lambda(|\phi^2|-1)^2 \end{aligned} \tag{5}$$

(the $*$ denotes the Hodge dual) reduces in the strong coupling limit to the Nambu-Goto action which govern the dynamics of the vortices [8].

On the other hand, writing the scalar field as $\phi = \rho e^{i\varphi}$ we can express the action S as

$$\begin{aligned} S = \int \frac{1}{2}F\wedge {}^*F + \frac{1}{2}(d\varphi + eA)\wedge {}^*(d\varphi + eA) + \\ + \frac{1}{2}d\rho \wedge {}^*(d\rho) - V(\rho). \end{aligned} \tag{6}$$

The equation of motion for A and φ which follows from this action,

$$d\,{}^*F = {}^*(d\varphi + eA)e, \tag{7}$$

can be solved by introducing a two-form field B whose field strength is the three-form field H:

$$d\varphi + eA \equiv {}^*(dB) \equiv {}^*H, \tag{8}$$

provided that

$$d\,{}^*H = eF \tag{9}$$

Then, using eq.(9), we can rewrite eq.(6) as

$$S \equiv= \int \frac{1}{2}F \wedge {}^*F + \frac{1}{2}H \wedge {}^*H + eB \wedge F + \\ + \frac{1}{2}d\rho \wedge {}^*(d\rho) - V(\rho). \tag{10}$$

In other words, there is a duality transformation connecting the scalar field φ with a Kalb-Ramond B field. So, in the strong coupling limit we have three equivalent classical actions: (2), (5) and (10).

By adding to the action (10) a coupling term with an *external* 2-form potential B^{ext} given by (3),

$$\delta S = eB^{ext} \wedge F, \tag{11}$$

we get an action equivalent to (1). If we now perform the duality transformation which transforms the 2-form field B into the scalar φ, this term becomes

$$e \int j \wedge A \equiv e \oint_C QA, \tag{12}$$

where $j_\mu(x) = Q \oint_C \delta(x-y) dy_\mu$ is a 1-form dual to the external 3-form vortex external source. Notice that we are generalizing the action (1) by including an arbitrary vortex, instead of a static vortex.

Thus, we conclude that the system consisting of a Maxwell field interacting with a a dynamical scalar field plus an external source –the term (12)–describes (at least in the strong coupling limit) the composite (1), and will probably present statistical transmutation at the quantum level.

In order to consider the quantum treatement of the preceding vortex excitations we will resort to the lattice formulation of the Maxwell-Higgs

system. There are several examples in the lattice field theory such that a change of variables in the partition function allows for a formulation of the theory in terms of the physical excitations. Banks, Kogut and Myerson [9] introduced a general tranformation which gives rise to a description of any abelian gauge theory with compact variables in terms of variables on the dual lattice associated with the topological excitations [10]. This was a generalization of the technique used by Jose, Kadanoff, Kirkpatrick and Nelson [11] to derive the Kosterlitz-Thouless [12] phase transition in terms of vortices for the $D = 2$ XY model. Given a D dimensional lattice theory with abelian compact variables on c_{k-1} cells ($k = 1$: sites and spin theory, $k = 2$: links and gauge theory, $k > 2$ hypergauge theory), by means of the $Banks - Kogut - Myerson$ transformation, one arrives to the '*topological*' expression of the partition function given by

$$Z_T \propto \sum_{\substack{{}^*\sigma \\ (\partial\, {}^*\sigma = 0)}} \exp[-2\pi^2 \beta \sum_{c_{D-k-1}} {}^*\sigma \hat{\Delta}\, {}^*\sigma\,], \tag{13}$$

where ${}^*\sigma$ denotes an integer variable attached to the c_{D-k-1} cells of the dual lattice. They correspond to the closed (due to the constraint $\partial\, {}^*\sigma = 0$) world "trajectories" of the $D - k - 2$ dimensional topological excitations. In four dimensions, one has world sheets for $k = 1$ and world trajectories for $k = 2$. The $\hat{\Delta}$ represents the propagator operator.

Let us now come back to the Maxwell-Higgs theory. For simplicity, we consider the limit $\lambda \to \infty$ which freezes the radial degree of freedom of the Higgs field.This is not a strong restriction, in fact it is known that the numerical results already obtained at $\lambda = 1$ are indistinguishable from the frozen case. Thus we get a compact dynamical scalar variable, i.e. $\varphi \in (-\pi, \pi]$ interacting with a non-compact gauge field $A_\mu(x) \in (-\infty, \infty)$ [2].

This model is known to possess two phases, namely, Higgs and Coulomb [13]. The Higgs phase supports the closed magnetic vortices.

The partition function for the Villain [14] form of the lattice action [3] is

[2]If one considers compact gauge field i.e. an angle $\theta_\mu(x)$ then, in addition to the Nielsen-Olesen vortices, there would occur Dirac magnetic monopoles [9] associated to this second compact variable in the *topological* expression of the partition function.

[3] We choose the Villain form instead of the ordinary Wilson form only for simplicity, with the Wilson action it is possible to repeat all that we do here.

given by

$$Z = \int (DA) \int (D\varphi) \sum_{n}$$
$$\exp[-\sum_{c_1} \frac{\beta}{2}(dA)^2 - \frac{\kappa}{2}(d\varphi - 2\pi n - A)^2], \tag{14}$$

where DA ($D\varphi$) denotes the integral over all link c_1 (site c_0) variables A (φ), $\beta = \frac{1}{e^2}$ is the gauge coupling constant, κ is the Higgs coupling constant and $n_(c_1)$ are integer variables defined at the lattice links c_1 i.e. $k = 1$. For this lattice model the propagator operator appearing in the *topological* representation (13) is given by $\hat{\Delta} = \frac{1}{\Box + m^2}$, where $m^2 = \frac{\kappa}{\beta}$ is the mass acquired by the gauge field due to the Higgs mechanism.

For $D=4$ the c_{D-k-1} are plaquettes c_2 and the partition function (14) can be expressed as a sum over closed surfaces on the dual lattice [15] which are the world sheets of closed string-like objects.Notice that $k = 1$ and therefore $D - k - 2 = 1$). These closed strings are obtained by intersecting the closed world sheets with a plane $t = constant$ and can be interpreted as the lattice version of the classical (continuum) Nielsen-Olesen magnetic vortices.

The partition function with an external current loop of the kind of (12), generated by a charge Qe, can be written in the topological representation as

$$Z_T[j_C] \quad \propto$$
$$\sum_{\substack{^*\sigma(*c_{D-k-1}) \\ (\partial\, ^*\sigma = 0)}} \exp[-2\pi^2\kappa \sum_{*c_{D-k-1}} {^*\sigma} \frac{1}{\Box + m^2} {^*\sigma}$$
$$-\frac{Q^2e^2}{2} \sum_{c_k} j_C \frac{1}{\Box + m^2} j_C$$
$$-2\pi i Qe \sum_{c_k} j_C \frac{1}{\Box + m^2} \partial\sigma$$
$$+2\pi i Qe \sum_{c_k} j_C \frac{1}{\Box} \partial\sigma]. \tag{15}$$

The first three terms in the exponent describe short range (Yukawa) interactions. The last long range term is a four-dimensional analogue of the

Gauss linking number for loops in three dimensions i.e. the linking number $\ell k(\sigma, j_C)$ of world sheets of the strings and the current j_C which appear in the Wilson loop.

If we consider the strong coupling limit $\kappa/\beta = m^2 \to \infty$ we get

$$Z_T[j_C] \propto \sum_{\substack{{}^*\sigma(*c_{D-k-1}) \\ (\partial {}^*\sigma = 0)}} \exp[-2\pi^2\beta \sum_{*c_{D-k-1}} {}^*\sigma^2\,] \exp[2\pi i Q e \sum_{c_k} \ell k(\sigma, j_C)] \qquad (16)$$

This is exactly the form of the Wilson loop average obtained in Ref [6] from the 3+1 generalization of the Polyakov [2] construction that includes a 4-dimensional Chern-Simons interaction.

$$\int B \wedge F, \qquad (17)$$

The continuous space-time form of this average is

$$Z = \sum_{\sigma} e^{-TS(\sigma)} \exp[-\frac{i}{4\pi^2} \oint_C dx \int_{\sigma_C} d\sigma(x')^*(d\frac{1}{|x-x'|^2})\,]. \qquad (18)$$

As in Ref.[2], the integral in the exponential is a topological number: the linking number $\ell k(\sigma, C)$ which measures the number of times the closed path C intersects the sheet σ in four dimensions.There is however an important difference between this expression obtained from the 3+1 extension of the Polyakov's construction and the lattice expression (16). As in the 2+1 case, the Polyakov's construction leads to a singular expression, since C must tend to be the border of σ, and some regularization process is needed, while in the lattice version the previous singularity has disappeared. From here, one can immediately prove the transmutation by following the same steps that in [6]. The main idea is that the action appearing in (18) leads after a Dirac quantization to a set of variables that behave as Pauli matrices and reproduce the propagator of the fermionic string.

Notice that in order to recover the specific coefficient appearing in (18) one has to suitable tune the value of Qe. This is in agreement with [4], where the Aharonov-Bohm effect produces the transmutation to a fermionic string only for certain values of Qe.

One could have followed a similar approach in the 2+1 case. Starting from the charge vortex pair, one could have studied the transition amplitude by

computing the Feynman path integral of a charged particle in the presence of a magnetic vortex. This computation would have led to the ordinary Gauss linking number and to the action already considered by Polyakov in ref[[2]].

So, we have proved that the stringlike excitations of the Maxwell-Higgs system in presence of an external charge undergo Fermi-Bose transmutation. It still remains to be understood if this kind of mechanism have some physical relevance, in particular if it may affect the behavior of the high temperature superconductors.

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